## MODELING OF THE THERMAL FIELD OF A WORKPIECE IN MILLING OF AUSTENITIC STEEL BY A FINGER-TYPE CUTTER

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The authors report results of modeling thermal fields by the method of finite elements under conditions of milling of 00H17N14M2A austenitic steel by a finger-type cutter. A solution of the problem of unsteady nonlinear heat conduction has made it possible to find the temperature distribution in the surface layers of a workpiece as a function of the cutting rate and the position of the tip of the cutter edge.

The released heat and the temperature in the cutting zone exert a great influence on the process of deformation of the material milled, the friction on the contact surfaces of the tool, and the residual stresses in the surface layers of the workpiece. A study of the regularities of heat transfer is especially important in processing of austenitic steels having unfavorable thermophysical characteristics.

The problem of determination of the temperature fields in the cutting zone is characterized, first, by the geometric nonlinearity of the domains of function determination, second, by the physical nonlinearity, i.e., the dependence of the thermophysical properties of the materials on temperature [1], and, third, by the joining of materials with different characteristics. Moreover, it is necessary to allow the unsteady mode of heat transfer and the nonlinearity of the boundary conditions. The only method to provide this is the method of finite elements. This method makes it possible to work out different degrees of detail of the solution in different regions of the object studied; use can be made of volumes that differ in size, configuration, and thermophysical properties.

The equations describing unsteady heat conduction in a solid body in the case of use of the method of finite elements are based on the equations of equilibrium of the heat flux. The conditions of heat conduction in a solid body that are idealized by the system of finite elements can be represented as the balance of the heat flux at the system nodes at any instant

$$Q_1 + Q_2 = Q_3 + Q_4 \,. \tag{1}$$

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This heat-balance equation holds for all internal nodes, but it must be modified for the surface nodes for the purpose of accounting for the boundary conditions.

The levels of the heat fluxes in Eq. (1) can be expressed in terms of the temperatures at the nodes of the element. The instantaneous heat flux in the material at any instant is found from the expression

$$q = \rho c (\Theta) V \frac{\partial \Theta}{\partial \tau} = C (\Theta) \frac{\partial \Theta}{\partial \tau}.$$
 (2)

If for a positive temperature gradient we assume that the heat flux is positive, then in passing through unit area in the direction  $\overrightarrow{n}$  it can be determined from the formula

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$$q_j = \sum_{n} \lambda_{jn} \left(\Theta\right) \frac{\partial \Theta}{\partial n} \,. \tag{3}$$

In the general case, Eq. (3) is written in the following form:

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} \lambda_{xx} & \lambda_{yx} & \lambda_{zx} \\ \lambda_{xy} & \lambda_{yy} & \lambda_{zy} \\ \lambda_{xz} & \lambda_{yz} & \lambda_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial \Theta}{\partial x} \\ \frac{\partial \Theta}{\partial y} \\ \frac{\partial \Theta}{\partial y} \\ \frac{\partial \Theta}{\partial z} \end{bmatrix}$$

Using the temperature characteristics determined by Eqs. (2) and (3), we can apply the Lagrange method to express the quantity of heat in (1) in terms of the temperatures at the nodal points [2]. The resulting matrix differential equation that describes equilibrium of the heat fluxes in a finite-element system containing m nodes has the form

$$\begin{bmatrix} C & (\Theta) \\ m \times m \end{bmatrix} \left\{ \stackrel{\circ}{\Theta} (\tau) \right\} + \begin{bmatrix} \Lambda & (\Theta) \\ m \times m \end{bmatrix} \left\{ \Theta & (\tau) \right\} = \left\{ Q & (\tau) \right\}.$$
(4)

Expression (4) represents a system of nonlinear equations from which the temperatures at the nodal points of the finite-element system are determined as functions of time. The matrices [C] and [ $\Lambda$ ] and the vector {*Q*} are known at each instant; therefore system (4) is solved by the methods of numerical integration.

The quantity of heat supplied to the region under consideration can be determined from calculations by using concepts of the theory of heat sources [3]. It is known that the heat released in the cutting zone is distributed among all the bodies participating in the heat transfer. Introducing the notion of resultant heat fluxes passing through contact surfaces of the tool, we can represent, with sufficiently high accuracy, the thermal regime of the workpiece, the cuttings, and the tool in the form of the equations

$$Q_{\rm w} = Q_{\rm def.w} + Q_{\rm fr.r} - Q_{\rm r}; \quad Q_{\rm cut} = Q_{\rm def.w} + Q_{\rm fr.f} - Q_{\rm f}; \quad Q_{\rm t} = Q_{\rm f} + Q_{\rm r}.$$

The fluxes toward the tool have the sign + and those from the tool have the sign -.

In considering the thermal fields of the workpiece, of primary importance are the conditions of heat transfer on the area of contact of the rear surface of the tool with the cutting surface. The forces on it occur as a result of elastoplastic interaction between these surfaces. Taking into account that the law of friction-force distribution along the wear area has the form [4]

$$\tau(h_{\rm r}) = 0.5\sigma_{\rm tem} \exp\left[-3\left(\frac{h_{\rm ri}}{h_{\rm r}}\right)^2\right],$$

we may assume that

$$F_{\rm fr,r} \approx 0.252 \sigma_{\rm tem} b h_{\rm r}, \quad q_{\rm fr,r} \approx 0.5 \sigma_{\rm tem} v \,.$$
<sup>(5)</sup>

It is easy to see that in (5) the known proposition of the absence of an influence of the processes on the rear surface on the processes in the zone of cutting formation is taken into account [3, 5].

The final goal of the investigations was to study the parameters of cold work hardening of the surface layer of a workpiece and to provide a certain level of it. Therefore we used small values of feed to the tooth



Fig. 1. Temperatures on the surface (coordinate *Y*) and in the body (coordinate *X*) of a workpiece made of 00H17N14M2A steel: a) for motion in the central part of the workpiece; b) as the workpiece edge is approached; c) on the workpiece edge; 1) v = 12.6 m/min; 2) 7.9; 3) 6.3; the motion is from right to left. *T*, <sup>o</sup>C; *X*, *Y*, mm.

that are comparable to the rounding radius of the tip of the cutter edge. With account for this we considered only the heat fluxes on the rear surface and simultaneously solved the contact problem.

According to the boundary condition of the fourth kind, when rod B moves (the tip of the cutter edge with account for the radius of its rounding) over half-space A (the workpiece surface), we obtain the coefficient characterizing the portion of the heat in the contacting body relative to which the source is immobile [6]:

$$b = \frac{1}{1 + \frac{3\lambda_{A}}{2\lambda_{B}}\sqrt{Pe_{A}Fo_{B}}},$$

$$Pe_{A} = \frac{v_{A} [m/sec] \times l_{A} [m]}{\omega_{A} [m^{2}/sec]}; Fo_{B} = \frac{\omega_{B} [m^{2}/sec] \times \tau_{B} [sec]}{l_{B} [m]}.$$

After transformations we obtain the coefficient that accounts for the quantity of heat arriving at the workpiece:

$$b^* = 1 - \frac{1}{1 + 1.5 \sqrt{\frac{\lambda c \rho}{\lambda_t c_t \rho_t}}}.$$

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Fig. 2. Change in the thermal field of a workpiece at different distances from the heat source (*X*, depth of the location of the point): 1) X = 0; 2) 0.025 mm; 3) 0.05; 4) 0.1.  $\tau$ , sec.

It is easy to note that the coefficient  $b^*$  for a specific pair of tool material-processed material is constant.

Thus, the heat flux  $q_{\text{fr.r}}^* = b^* q_{\text{fr.r}}$  arrives at the workpiece.

This procedure was applied to calculations of the heat fluxes and the temperatures in the zone of milling of 00H17N14M2A austenitic steel by a finger-type cutter made of SW7M high-speed steel with the number of teeth z = 4 in the range of cutting rates of 6.3–12.6 m/min with the feed 3.2 mm/min.

In modeling by the method of finite elements, we employed the following initial data: the total number and the coordinates of nodal points; the type of heat transfer (unsteady); the initial temperature (room temperature); the number of time steps of solution (up to 1200), the increment in a time step (0.0015 sec); the boundary conditions, namely, the heat fluxes at the nodes; the properties of the processed material, namely, the density, thermal conductivity, and specific heat as functions of temperature. The heat-conduction matrix was re-formed at each successive step.

In performing the calculations, we considered two cases: the motion of the cutter tooth over the workpiece surface and the change in the thermal field in the workpiece upon passage of the tooth. In so doing, the density of the heat flux on the workpiece surface was considered as a boundary function, changing with time, in relation to a certain node.

In analyzing the calculation results, we established (Fig. 1) that in the range of rates considered the heat flux executes a leading motion in front of the edge tip, which corresponds to low Péclet numbers (Pe < 2). The temperature on the workpiece surface increases in proportion to the cutting rate and the heat-flux density, as stipulated by the well-known Fourier heat-conduction equation [1, 5]. As the heat source approaches the edge of the workpiece, the heat-removal conditions worsen and a temperature increase of about 1.5-fold is observed. The highest temperatures are concentrated in thin surface layers of the workpiece (a depth of up to 0.05 mm) and then they decrease sharply.

Upon passage of the cutter edge the temperature in the surface layer decreases sharply and this occurs most intensely in the thin surface layers (Fig. 2). These results can easily be explained if the motion of the cutter tip over the workpiece surface is represented as that of a point source of heat through a half-space [1].

Use of the methods of regression analysis allowed us to determine a formula for calculation of the thermal field in the body of the workpiece:

$$\Theta = \exp(4.55 + 0.075v - 0.336Y - 4.048X).$$

The multiple-correlation coefficient for this equation is 0.93.

To check the reliability of the results obtained, we conducted experimental studies of milling of the steel investigated. Based on measurements by the natural-thermocouple method, we obtained a formula for calculating the mean temperature in the cutting zone:

$$\Theta = 135f^{0.235}v^{0.214}.$$

The experimentally measured temperatures turned out to be approximately 20–25% higher than the calculated ones. This is quite explicable since the natural-thermocouple method registers a mean temperature in the cutting zone that is always higher than the temperature on the workpiece surface [3].

Thus, use of the method of finite elements allows calculation of the thermal fields under conditions of unsteady nonlinear heat conduction, for instance, in milling. The temperature on the surface of a workpiece of 00H17N14M2A steel in milling attains 400°C but under conditions of hindered heat removal it is 600°C. The depth of occurrence of the layers most heated does not exceed 0.05 mm.

## NOTATION

 $Q_1$ , quantity of heat in the elements adjacent to the node;  $Q_2$ , quantity of heat removed from the elements adjacent to the node due to internal heat conduction;  $Q_3$ , quantity of heat transferred to the node from the external source;  $Q_4$ , quantity of heat generated in the elements adjacent to the node;  $C(\Theta)$ , mass heat capacity of the material; V, volume of the material;  $\vec{n}$ , direction of the temperature gradient;  $\lambda_{jn}$ , symmetric positive-definite tensor of heat conduction for the investigated material; x, y, z, coordinates;  $\lambda_{xx}$ ,  $\lambda_{xy}$ , ...,  $\lambda_{zz}$ , thermal conductivities in the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the direction of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of the corresponding coordinate axes;  $[C(\Theta)]$ , heat-capacity mammetric positive-definite tensor of tensor of tensor positive-defini

trix of the system;  $\{\Theta(\tau)\}$ , vector of the temperature derivatives with respect to time at the nodal points at the instant  $\tau$ ;  $[\Lambda(\Theta)]$ , heat-conduction matrix of the system, which depends on the temperature  $\Theta$ ;  $\{\Theta(\tau)\}$ ,

vector of the temperatures at the nodal points at the instant  $\tau$ ;  $\{Q(\tau)\}$ , vector of the external heat sources at the instant  $\tau$ ;  $Q_w$ ,  $Q_{cut}$ , and  $Q_t$ , quantity of heat arriving at the workpiece, at the cuttings, and at the tool, respectively;  $Q_{def.w}$ , quantity of deformation-induced heat released into the workpiece;  $Q_{def.cut}$ , quantity of deformation-induced heat released into the cuttings;  $Q_{\rm fr,r}$ , quantity of heat due to the friction between the cutting edge and the workpiece; Q<sub>fr.f</sub>, quantity of heat due to the friction between the front surface of the cutting edge and the cuttings;  $Q_r$ , resultant heat flux produced by the heat transfer on the area of contact between the cutting surface and the rear surface of the tool;  $Q_{\rm f}$ , resultant heat flux produced by the heat transfer on the area of contact of the above-cutter side of the cuttings with the front surface of the tool;  $F_{\rm fr,r}$ , friction force on the rear surface;  $\sigma_{\text{tem}}$ , temporal resistance of the processed material; b, width of the contact area;  $h_{\text{r}}$ , width of the wear area on the rear surface;  $h_{r,i}$ , running coordinate of the point on the rear surface;  $q_{fr,r}$ , density of the heat flux on the rear surface of the cutter edge;  $q_{\text{fr.r.}}^*$ , density of the heat flux toward the workpiece on the side of the rear surface; Pe<sub>A</sub> and Fo<sub>B</sub>, Péclet number for body A and Fourier number for body B; v, source velocity (cutting rate); l, characteristic dimension of the source;  $\partial \Theta / \partial n$ , temperature gradient in the direction *n*;  $\omega$ , thermal diffusivity;  $\lambda$ , thermal conductivity;  $\rho$ , density; *c*, volumetric heat capacity;  $b^*$ , coefficient accounting for the quantity of heat coming into the workpiece; Y, distance between the edge tip and the considered point in the direction opposite to the direction of the cutting rate; X, depth of occurrence of the point relative to the workpiece surface; f, feed. Subscripts: w, workpiece; cut, cuttings; t, tool; def, deformation; fr, friction; f, front; r, rear.

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